MS UNDER REVIEW

The Independence of Language and Number

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It has been argued that the discrete infinite character of numerical cognition is bootstrapped or inferred from language in development (Bloom 1994, 2000). Others (Hurford, 1975;1987) have argued that adult numerical cognition is entirely derivative of language. We argue, to the contrary, that language and number are independent mental faculties and furthermore that the notion of cross-faculty bootstrapping is conceptually implausible. We further present empirical evidence of a double-dissociation between the two faculties, suggesting that number is not derived from grammar during development or in the adult state. In spite of the independence of the number and language faculties, we show that in the grammar of the count routine there is evidence of properties particular to the number faculty, suggesting a *lexical* interface between the two faculties.

0. Introduction

The goal of many researchers in cognitive science has been to discover the fundamental principles of mind that underlie systems of knowledge and to determine the extent to which such systems may or may not constitute distinct cognitive domains. As part of this inquiry, knowledge of discrete numbers and its relationship to language have been discussed. Within this discussion it has been claimed that number is derivative of language in its structural properties and, as a consequence, is dependent on language for its developmental emergence. In this paper we argue against this position and argue instead that number constitutes a distinct mental faculty which is not and could not be derivative of language. We suggest that these two independent faculties interface in ways reminiscent of the interface between grammar and spatial cognition suggested in Landau & Jackendoff, 1993.

In our consideration of these issues, we will focus on the properties of the number faculty manifested in the counting process and its representation, as elucidated in Gallistel & Gelman, 1992, 2000, and Gelman & Gallistel, 1978. We address the mental representation of the counting (positive, natural) numbers only. We assume that the number faculty can manifest itself in other processes or computations as well, including estimation, addition, subtraction and judgements of equivalence and ordering. However, counting is a process in which (at least) numerical, grammatical and pragmatic aspects of cognition interface. As a consequence, it offers an especially good window through which to examine the interface between language and number and to evaluate proposed relations between the two.

Recent proposals, (Bloom, 1994, 2000) and (Hurford, 1978, 1987), argue that the number faculty is dependent on the language faculty to various degrees. Hurford's position is that with the correct formulation of recursive phrase structure rules, well-formedness conditions and particular assumptions about semantic interpretation, no reference to an independent module of numerical cognition is necessary. Rather, numerical cognition can be derived from grammatical knowledge in combination with the general ability to process objects and collections of objects.

> . . . the number faculty largely emerges through the interaction of central features of the language faculty with other cognitive capacities relating to the recognition and manipulation of concrete objects and collections. The relevant features of the language faculty include the pairing of words with concepts by the linguistic sign (à la Saussure) and highly recursive syntax.

> It is therefore not necessary to postulate an autonomous 'faculty of number' as a separate module of mind. (Hurford, 1987, p. 3.)

In Bloom's view, the number faculty receives its discretely infinite character from the language of counting through a bootstrapping process (Bloom, 1994) or by inference (Bloom, 2000). Here we understand the property of "discrete infinity" of the numerical and grammatical domains to be the possibility of combining some set of symbols in such a way that a potentially infinite set of combinations of these symbols may be generated. Illustrations of this property in number and grammar are given in section 2. Bloom argues that while this property holds from the very beginning in grammar, it does not exist initially in the numerical domain. Rather it must be bootstrapped or inferred out of the grammar of the count routine into number. We will discuss bootstrapping in section 3. By "count routine" we mean the language used when counting things in the standard way (1 cat…2 cats…3 cats…, or 1,2,3 cats [as a child points to each item]). Bloom makes this proposal to explain Wynn's (Wynn, 1990, 1992b) finding that children apparently do not begin to count reliably until they are approximately 3 and a half. In Bloom's view, this delay results from the fact that although children can produce the count routine early on, they nonetheless lack the knowledge that the count routine corresponds to a number system which is infinite in principle. He points out that the count routine uses grammar, which we know is discretely infinite through processes like relativization (for syntax) and compounding (for morphology). Hence, the fact that grammar (a discretely infinite system) is used to express number (a system that in its initial state is only *potentially* discretely non-finite, in our reading of Bloom’s claim) creates a context which allows bootstrapping of the property of discrete infinity from grammar into number.

> . . . in the course of development, children ‘bootstrap’ a generative understanding of number out of the productive syntactic and morphological structures available in the [linguistic] counting system (Bloom, 1994, 186.)

Or, in his later formulation

> Under this view, it is not that somehow children know that there is an infinity of numbers and infer that you can always produce a larger number word. Instead, they learn that one can always produce a larger number word and infer that there must therefore be an infinity of numbers. (Bloom, 2000, 238.)

We will suggest that the interaction between number and grammar is best understood as resulting from the interface between two independent cognitive domains. From this perspective, Hurford’s “derivative” view of number appears improbable given the evidence for the independence of number and grammar that we will present. Bloom’s “bootstrapping” and "inference" proposals, however, are more plausible because they assumes the existence of the two independent domains. Furthermore, Bloom’s proposal assumes a kind of interaction between these two domains, as do we. We depart from Bloom’s proposals, however, because he proposes that the <u>property</u> of discrete infinity in one symbolic system (grammar) can be transferred to another symbolic system (number) without transferring the symbols and computations which generate this property. Because the symbols and computations are logically prior to the property, transferring the property without the symbols and computations cannot follow. We, too, suggest that there is interaction between the two domains, however we assume that it is number which constrains grammar in that properties specific to the counting process are reflected as grammatical properties in the linguistic count routine.

The structure of the paper is as follows. In section 1, we present our view of the cognitive domains of number and language. In section 2, we review basic properties of recursive systems in order to make our discussion of Bloom's proposal precise and present a conceptual argument against the bootstrapping of discrete infinity, based on the fact that the property of discrete infinity results from the rule system it characterizes and thus cannot be bootstrapped or, divorced from the rule system. In section 3, we review evidence from developmental disorders that demonstrate both the ontogenetic as well as adult-state independence of the faculties of number and grammar, casting doubt on the Hurford's proposal that number and grammar are one and the same and also on Bloom's proposal that number gets it discrete infinite character from grammar in the course of cognitive development. In section 4, we discuss, in the light of the empirical evidence presented in section 3, what the cognitive architecture in question must be like and we discuss the role which bootstrapping theories may play within this architecture. In this section we also discuss the language-particular counting grammar of English, and show that, contrary to the systems of Bloom and Hurford, the grammar of the English count routine is not characterized by potentially discretely infinite grammatical constructions, and consequently presents no real opportunity either for the Number faculty to depend on grammar in the adult state, nor for discrete infinity to be bootstrapped or inferred into the Number faculty. In section 5 we situate the language-number interface in the lexicon, and discuss instances in which properties of numerical representations are reflected in the grammar of the linguistic count routine.

1. Number and Grammar as Computational Domains

Let us begin by defining the mental domains to which we will refer throughout. Grammar and Number are both domains which are capable of generating infinitely many distinct objects. There is evidence that these domains support representations of various complexities, which suggests that they have an implicit recursive definition of an infinite representational space, the use of which is limited by factors irrelevant to the definition of that space (memory limitations, attention, sleep, life span, etc.). In other words, both Grammar and Number produce objects which in practice are finite. However, there does not appear to exist any conceptual impediment, internal to Grammar or Number, to increasing the complexity of these objects infinitely.

Grammar is a domain which can produce infinitely many objects associated with gestures of some kind. There appear to be no linguistic limitations on the length of a sentence. When limitations are imposed, they are imposed by non-linguistic factors of the kind just described. Number is a domain in which infinitely many structures are associated with different numerosities (sets of one or more objects). This infinite number domain is only limited by factors irrelevant to numerical cognition, as with grammar. Counting is the process by which some array of objects is paired with a corresponding representation in the numerical domain. Following Gelman & Gallistel, 1978, we will refer to these representations in the numerical domain of the mind as *numerons*. Linguistic counting is the process by which a representation in the grammatical domain is paired with a representation in the numerical domain. The linguistic representation may or may not be expressed as a gesture (vocal, manual, orthographic or otherwise). When such a number/grammar pairing is made, we refer to the lexical item as a *numerlog*, again following Gelman and Gallistel. Counting in the numerical domain can proceed independently of its grammatical representation. Infinitely many linguistic expressions can be paired with infinitely many numerosities.

We will claim that the psychological mechanisms responsible for what we have defined as counting, as opposed to linguistic counting, are independent of grammar and that the mechanisms responsible for the discretely infinite character of number are not and could not be bootstrapped out of grammar.

2. Recursion

Language and number both have the property of discrete infinity. With respect to language, Chomsky (1955) observed that this property could be formalized using recursive function theory. In a recursive system, infinitely many structures may be built up from finitely many discrete units because the rules or principles of the system allow structures to recur within one another (proper inclusion) without limit.

In language, recursion is pervasive. Recursion is possible through nearly every major phrasal category. For example, a simple phrase structure grammar such as (1) allows recursion through AdjP (Adjective Phrase) to generate infinitely many constructions like those in (2).

(1) *A simple phrase structure grammar*
NP → Art N'
N' → (AdjP) N
AdjP → (Adv) AdjP
AdjP → Adj
Art → a
Adv → very
Adj → happy
N → farmer

(2) a farmer
a happy farmer
a very happy farmer
a very very happy farmer
a very very very happy farmer
. . .

With respect to counting, the property of discrete infinity might be represented in terms of a successor function *s,* such that for any number n, s(n) is $n+1$. Thus, (3) builds (unary) recursive structures, each properly included in its predecessor, as in (4).

(3) *The Successor Function*

For any number n, s(n) is $n+1$

(4)

$$\begin{array}{c} \dots \\ | \\ s(4)=5 \\ | \\ s(3)=4 \\ | \\ s(2)=3 \\ | \\ s(1)=2 \\ | \\ 1 \end{array}$$

Notice that the property of discrete infinity only exists in the numerical and grammatical domains by virtue of the particular symbols and computations that range over those symbols, as in (3) and (4). These computations have the form of a recursive function which builds primitives into structures. Hence the existence of the property of discrete infinity <u>arises from</u> the particular rule systems that it characterizes.

To reiterate, discrete infinity is <u>property</u> of a system of rules. It emerges because the rules of the system have a particular character, namely, their form is that of a recursive function which builds primitives into structures. Hence, the existence of such a rule system is logically prior to the emergence of the property of discrete infinity. While there has been some success in the cognitive sciences in characterizing such systems in a number of cognitive domains (for instance, vision, language, number and music), it is difficult to imagine what it could mean to infuse a system which lacks the property of discrete infinity, with the property of discrete infinity without changing the primitives or functions of that system. Indeed, many objects in the natural world share properties (any two organs of the human body, for instance), but such observations do not generally lead us to think that there is a <u>developmental</u> interdependence of some kind between them.

Nonetheless, in Bloom (1994, 2000), the observation that number and language share the property of discrete infinity is presumed to make a case for the claim that number emerges from language ontogenetically, despite the intractability of implementation.

3. Dissociations Of Language And Number in Ontogeny

Thusfar, we have presented the conceptual argument that discrete infinity cannot be bootstrapped from grammar into number because it is a property of rule systems, which could not plausibly "move" unless the rules "moved". This suggests that Bloom's proposal is mistaken. But his proposal, as well as Hurford's that Number depends entirely on Grammar, raise empirical as well as conceptual questions.

The first empirical question concerns our claim that Number and Grammar are independent mental faculties: if Number and Grammar are independent mental faculties should it be possible for one to develop in the absence of the other? To answer that question, we will present evidence from cases of mentally retarded individuals who demonstrate intact grammatical development yet possess limited or no ability to calculate and little grasp of basic counting principles. The second empirical question is: if number depends on grammar in the way that Hurford suggests, what kind of numerical cognition will be possible for adults who lack grammar? The third empirical question is: if Number derives its discrete infinite character from Grammar during development, as Bloom suggests, how will numerical cognition develop in those who lack grammar? To answer the second and third questions we present evidence from hearing-impaired adults, who have developed no Grammar, in the sense of natural human languages, but nevertheless are able to use Number.

One case illustrating a developmental dissociation between Number and Grammar is Rick (Curtiss, 1988a, 1988b), a mentally retarded teenager, who suffered anoxia at birth, had severe motor disabilities, which made it impossible for him to sit erect, stand, or walk, and who spent all of his childhood and adolescence in a state hospital for the severely retarded. Despite pervasive retardation, Rick's language embodied a rich syntax, with both DP and VP recursion, as illustrated in (5) - (9).

(5) She's the one that walks back and forth to school

(6) It's what I do

(7) I find pictures that are gone

(8) She looks like she has blonde hair

(9) You already got it working

Despite his well-developed knowledge of grammar, Rick could count by rote only to 20, could not perform any arithmetic operations, could not tell time, did not know his age, and could apprehend numerosity differences only between sets no larger than 3. Moreover, exemplifying the contrast between his linguistic and number knowledge, Rick used number words and expressions frequently, but knew little about their meaning, as illustrated in (10).

(10) Investigator: Who gets up first in the morning?

Rick: I do. And Cindy wakes up third.

I: Third? Who gets up second?

R: Nobody.

An even stronger developmental dissociation between grammar and number is seen in the case of Antony (Curtiss, 1988a, 1988b; Curtiss & Yamada, 1981), whose language indicated normal grammatical development but whose cognitive performance evidenced no number knowledge at all. At the age of 7 years, Antony functioned cognitively at a level of approximately 18-24 months. He could not dress himself, could not draw representationally, and more pertinently, could not count, could not demonstrate an understanding of the concepts 'more' or 'same,' nor even an ability to differentiate sets of 2 items from sets of 3, 4, or 5 items. Yet, throughout his developmental history of marked retardation, he showed surprising linguistic growth, reportedly producing 2 and 3-word utterances at age 2, and full sentences at age 3. Antony's language clearly possessed the properties of discrete infinity and recursive enumeration, as illustrated by sentences containing small clauses, embedded participial clauses, infinitival clauses, WH-complement clauses, and relative clauses, shown in (11) - (15).

(11) Jeni, will you help me draw pictures of Susie? [small clause]

(12) I don't want Bonnie coming in here [participial clause]

(13) He wants to chase the cat [infinitival clause]

(14) I don't know who he gots [WH complement with object-extraction]

(15) a stick, that we hit peoples with [relative clause, with object extraction]

A third individual showing the same dissociation is Laura (Curtiss, 1988a, 1988b; Yamada, 1990), who was studied at the end of her adolescence, and who showed, perhaps, the most sophisticated grammatical knowledge of the three, and thus perhaps the most fully developed language faculty alongside a markedly undeveloped number faculty. Laura could rote count into the teens, but despite years of special schooling, did not know the basic counting principles. She would often assign two numerlogs to the same item, at times count the same item several times, and often made errors counting arrays of only 3 or 4 items. In striking contrast, Laura had a fully mature syntax, .as illustrated in (16) - (20).

(16) Did you hear about me not going to this school up in Altadena?

(17) She does paintings, this really good friend of the kids who I went to school with last year and really loved.

(18) He was saying that I lost my battery powered watch that I loved

(19) It makes me feel sad because they had to leave

(20) I'm very good friends of a girl that cuts (...)'s hair, that I'm working with

Her retardation (e.g., WISC IQ 41 - 48 at different testings[1]) was manifested in many lexical inaccuracies and difficulties. While Laura had a large lexicon, many of its entries were underspecified with respect to denotative content; and in particular, although her lexicon included many number-related entries, their numerical denotation was not appreciated. (21) - (26) exemplify her misuse of such expressions.

(21) He's my third principal I've had since I've been here [untrue]

(22) Oh, frack, we finally got that new Mexican ‘cause his flights came in Wednesday month

(23) And I told the head leader they’re not sure if they’re gonna set if for, for eight, eighth, our time which will be as [abrupt pause] our time and, the girl arrives where it’s one, which is in school right now.

(24) I was like 15 or 19 when I started moving out o’home, so now I’m like 15 now, and I can go

(25) a good friend’s second friend

(26) J: How many nights did you stay there? (at a hotel with her family)

L: Oh, about 4 out of 1

All three of the above cases display a striking disparity between knowledge of Number and knowledge of Grammar, supporting a position which holds that these are developmentally autonomous faculties. While this evidence is relevant to our first empirical question in that apparently Grammar can develop in the absence of Number, it does not directly contradict the claims of either Bloom or Hurford, who do not suggest that Grammar depends on Number.

Evidence that Number can develop in the absence of Grammar, however, contradicts the core of both Bloom’s and Hurford’s arguments and supports our view that the two domains are independent in the adult state and in development. Let us, then, attempt to answer our second and third empirical questions regarding the adult state and the developmental course of the Number faculty in the absence of Grammar. In an attempt to shed light on this question, we will examine cases of individuals who appear to have fully developed Number faculties; i.e., knowledge of how numbers work -- knowing how to perform arithmetic operations at will, from counting to multiplication -- while not possessing language. A number of such cases are documented.

Chelsea is one individual who shows such a dissociation . A hearing-impaired woman who grew up in a then small, rural community, without learning any natural language, Chelsea was "discovered" in her early thirties and has been the subject of much habilitation, instruction, and study (Curtiss, 1988b, 1994, 1996; Dronkers, 1987; Glusker, 1987). With aids, her hearing falls within the normal range, and she now possesses a substantial spoken, sign, and written vocabulary, which continues to increase. However, after 13 years of language instruction and exposure, she still does not possess the rudiments of natural language grammar, such as knowledge of phrasal or clausal structure, recursive syntactic rules, morphological rules of word formation,[2] even syntactic properties such as the C-selection features or theta structure requirements of words long in her productive vocabulary. Note, for example, some sample utterances below, all constructed of words which have been in her productive vocabulary for years:

(27) Missy girl same both girl (1987) [comparing the gender of 2 animals]

(28) Cat chasing cat (1992) [She had been asked: *What is the cat chasing?* Answer: A dog.]

(29) Fort Bragg Fort Bragg L.A. your (1992) [a comment about where we were each from]

(30) P. broken. Nervous see P. (1995) [P's car had broken down. Chelsea could see that P was nervous]

(31) Me pay money me grass money G (1995) [C. paid G money, situation unclear]

The persistent absence of the basic structural principles of grammar suggests an inability of the language faculty to develop or be instantiated in relevant respects, at this stage in her cognitive/neurological life. Yet, despite the absence of the properties of language which have been suggested as those relevant for triggering the development of the number faculty, Chelsea can perform all basic mathematical operations. She can perform such operations in her head as well as on paper, understands and uses money correctly, even balances a checkbook! (Glusker, p.c.). As far as can be determined, she acquired all of this knowledge as an adult. What is more, she uses number words and expressions correctly, as illustrated in (32) - (36), can tell time, can talk about specific times, e.g., (35), and can talk about numbers and money, such as in discussing the cost of things, as in (36).

(32) (Pretending she's the tester)) How many apples? Seven apples

(33) (C is staying in a house with 3 bathrooms, but has seen only the 2 upstairs. She is downstairs, speaking to J, one of the inhabitants)

C: I go bathroom (C turns away and starts to go upstairs. J calls after her)

J: There's a bathroom down here! (C turns around. J. points.)

C: Three. Three bathroom.

(34) Baby. Have 2. (re my having two children)

(35) Go work 8:30?

(36) (Re needing to buy a new battery for her hearing aid.)

C: Change. Throw away. Battery no good. Pay less.

S: How much do they cost?

C: Three dollar. Pay less. Fifty cent. (She only paid $2.50.)

Most importantly, in performing calculations on or reading numbers in the hundreds or thousands, she uses number words and expressions that require and embody numerical cognition. Thus, Chelsea possesses numerical cognition without a linguistic grammar.

Other examples in the literature argue against the need for knowledge of any aspect of natural language for the acquisition of number knowledge. (Schaller, 1991) reports several cases of deaf adults without any exposure to or knowledge of language at all, who have mastered counting and simple arithmetic, some who had invented their own means of representing integers up to ten as well as multiples of ten.

From a description of the quick mastery of addition (via overt instruction) by one young adult with no language,

> I drew an addition sign between two 1s and placed a 2 underneath. I wrote 1 + 1 + 1 with a 3 under it, then four 1s, and so on. I explained addition by placing the corresponding number of crayons next to each numeral. He became very animated, and I introduced him to an equal sign to complete the equations. Three

> minutes later the crayons were unnecessary. He had gotten it. (Schaller, 1991, p. 37.)

Her description continues,

> Ildefonso could add and subtract using numerals one through nine; he was ready for ten. Could I teach more complicated arithmetic without language? .... I counted nine crayons in front of Ildefonso, added one, tied the bundle with a rubber band and placed it on a piece of paper. Below it I wrote 1 0. Then I wrote 1 through 10 and pointed to the bundle for ten. I continued with 11, 12, 13, 14, 15 ,adding crayons. I repeated the lesson. ... I made two bundles and placed them together, writing 2 0 below them. Slowly, I wrote 1 through 20 and placed the corresponding crayons above each numeral. Ildefonso studied what I had written -- and got it without any more help (Schaller, 1991, p. 60-61.)

Another quote by Schaller describes the means of counting and representing number invented and shared by several deaf men who knew no spoken or sign language:

> They counted by holding their palms out and extending their fingers, then they pushed their hands slightly forward to indicate ten. They turned their hands around, palms facing in, and pushed forward for twenty, and so forth until the counter lost track. (pp. 183-184.)

In Galvan's description of his work with a languageless twenty-year-old deaf Filipino subject (cited in Schaller), he reports that he readily taught his student numbers so that he could read the train and bus schedules, and that this student showed great interest in money and cards, both of which became useful methods for teaching him more about numbers and arithmetic operations.

Now let us return to the empirical questions raised at the beginning of this section. The second was, "What kind of numerical cognition will be possible for people who lack Grammar, if Number depends crucially on Grammar in the adult state?", as Hurford proposes. Hurford's account would seem to predict that either a complete absence of numerical competence or at least a severe impediment of numerical competence should be observed, given the crucial dependence of Number on Grammar. However, we observe in Chelsea's case that her numerical cognition appears completely adult-like (and perhaps even advanced to those who have trouble balancing a check book). Consequently, Hurford's proposal seems highly implausible.

The third empirical question, raised at the beginning of the section was, "If Number derives its discrete infinite character from Grammar during development, as Bloom suggests, how will numerical cognition develop in those who lack Grammar? " The answer, from Bloom's perspective, must certainly be that it should not be possible for numerical cognition to develop in the absence of Grammar. And yet Chelsea and the other hearing impaired subjects mentioned appear to be individuals whose cognitive development did not include discrete infinity in the grammatical domain, and who, nonetheless display command of number faculties characterized by discrete infinity. Clearly, his proposal must be mistaken as well.[3]

Finally, the children whose mental retardation resulted in their developing normal (discretely infinite) grammatical abilities, in the absence of discretely infinite numerical abilities, strengthens our contention that Number and Grammar are independent both in the adult state and in development.

4. Modularity and Bootstrapping Theories

We have made the argument, for conceptual reasons, that bootstrapping of a property, as Bloom proposes, should be impossible, and that in the crucial empirical cases just presented, such bootstrapping appears to have been unnecessary for the development of normal numerical cognition. Given the evidence presented, how shall we characterize these apparently independent mental faculties?

4.1 Modularity

Fodor's (Fodor, 1983) treatise on modularity characterizes language and the perceptual systems as "the mental modules," contrasting these with "the central processes" or "thinking." For Fodor, modules differ from general reasoning in that, among other things, they are subject to characteristic and specific breakdown, are domain-specific and are informationally encapsulated. We have just seen evidence of the "characteristic and specific breakdown" in the double dissociation of Number and Grammar, but what about the "domain-specificity" and "informational encapsulation" of Number and Grammar? Fodor's discussion of domain-specificity and informational encapsulation is relevant to our proposal because it places limits on the contact among mental faculties. If mental faculties are limited in the ways that they can communicate with one another, then logically they will be limited in their abilities to participate in bootstrapping operations.

By domain-specific and informationally encapsulated, Fodor means that the operations used to process information of one domain are not used to process information of another domain. Thus, only the visual system functions to provide information about three-dimensional spatial layout, shape, trajectory, and color from retinal arrays. Only the language system interprets structural linguistic information regarding phonotactics, syntactic movement and pronominal coreference. Similarly, domain-specific, numerical information about properties of sets of objects are computed only by the number faculty. This is relevant to our proposal in that bootstrapping between two domains should be impossible if the operations of one domain cannot make use of information in another domain.

Jackendoff (Jackendoff, 1987, 1992) proposes a refinement of Fodor's (1983) formulation, taking the domain specificity of mental modules to be "a consequence of the formal representations they operate on," so that "modularity of processing is determined precisely by the forms of mental representation being processed" (1993: vii). This construal of faculty-specificity is consistent with much work in neuroscience, which has concluded that the nature of sense data determines the form of its cognitive representation as well as the computations which range over them (cf. (Bradley, Maxwell, Andersen, Banks, & Shenoy, 1996; Rauschecker, Tian, & Hauser, 1995; Weinberger, 1995) Taken together, these lines of thought suggest that mental faculties are structured to process particular types of sense data (and not others), represent that data using mental symbols particular to the faculty in question (and not those of other faculties) and perform mental computations over those representations which are also particular to the mental faculty in question (and cannot range over representations of other mental faculties). The apparent autonomy observed in the development of Number and Grammar in the previous section supports this view of autonomous mental faculties. Such a view implies that it is highly unlikely that the computations of one domain, such as Number, could process the symbols of another domain, such as Grammar.

Let us now attempt to consider somewhat precisely what it must mean for a property such as discrete infinity to be bootstrapped from one domain to another. We will take advantage of Jackendoff's representation-based notion of modularity and work by (Keenan & Stabler, 1994) on general grammars to formalize our definitions and claims regarding bootstrapping. We take a mental faculty to be a fourtuple $MF = <V, Cat, Lex, F>$. V is a set of terminal vocabulary items (e.g., V_1 includes words like "building" in grammar, V_2 includes shapes such as a three-dimensional cube in vision, and V_3 would include the numeron "7" in number). *Cat* is a set of categories: Cat_1 is {V, N, P, Adj, Adv ...} in the case of grammar, Cat_2 is {edge, angle, color, hue ...} in the case of vision and Cat_3 is {natural numbers, whole numbers, rational numbers} in the case of number. *Lex* is a set of paired expressions $<V, Cat>$, and F is the set of structure building (partial) functions which builds smaller structures into larger structures. Thus, the set of expressions defined by such a system is the lexicon *Lex* plus everything which can be built using the generating functions F. The resulting set of expressions is the closure of the lexicon under the structure building functions, $E(MF) = CL(Lex, F)$.

This formal system captures the property of domain-specificity for mental faculties because the structure-building functions F_1 can only operate on the lexicon Lex_1 of a single mental faculty. Hence, the successor function operates on a numerical representation to render its successor.[4] Thus, there is always a next natural number.

However, it makes no sense to speak of the successor function, or of any other operation or principle peculiar to the number faculty, as operating on, for example, a syntactic string to compute its meaning, structure, or even its length.[5]

In addition, the more formal definition of mental faculty provided above also captures the property of encapsulation, since it does not allow structures in $E(MF_1)$ (all the expressions of MF_1) to be built by applying F_1 to Lex_2. In other words, because the input values in different faculties are different types of objects (morphological features and structural representations in the case of language, sets and numerons in the case of number), they cannot be used by a single "general-purpose" function to build structure. That is, there is no operation in F_1 (or anywhere else) that can use elements of Lex_2 to construct expressions of MF_1; these operations are faculty-specific, and the faculties informationally encapsulated. This "faculty-specific" view of numerical and grammatical computations seems most plausible to us in the light of the conceptual and empirical arguments presented.

Notice, too, that although the set of expressions built in this way is discrete and infinite for both number and language, discrete infinity is a property of the respective rule systems. Concretely, then, the claim that this property is in some way "extracted" across faculties must be a claim about "extracting" *rules and representations* across faculties. We see no other way to construe the notion that discrete infinity moves across faculties. Thus, in addition to the prima facie implausibility of bootstrapping a property across domains, based on their mere co-occurrence, mentioned in section 2, cross-faculty bootstrapping is made even less plausible by the fact that rules and representations of one faculty are not commensurable with rules and representations of another by virtue of faculty-specificity and informational encapsulation. Thus, even if such an operation existed, the “donor” faculty could not provide the other with useable rules or representations.

4.2 Bootstrapping Theories

Bootstrapping theories attempt to explain gaps between inputs and outputs by positing a particular “coordination” of information across mental sub-domains, but never across mental faculties. Bootstrapping theories, Pinker (1987) notes, are concerned with the observation that “there is no direct relation between the types of information in the input and the types of information in the output.” Thus, a child receives linguistic input in the form of a continuous string of acoustic energy, but the output is a complex representation involving word meaning, morphological and phonological form, word formation rules, syntactic categories, and complex grammatical relations defined on syntactic trees.

Cutler (1994), Jusczyk (1993) and Mehler et al., (1988) and others suggest a prosodic theory of bootstrapping which posits that a child takes advantage of prosodic information in the stream of speech in order to determine which sounds form words and which sound strings form phrases and clauses. (Grimshaw, 1981) and (Pinker, 1984) propose semantic theories of bootstrapping which take children's knowledge of word meaning to infer its syntactic properties. (Gleitman, 1990) and (Fisher, Hall, Rakowitz, & Gleitman, 1994) provide evidence for syntactic bootstrapping in which the child’s knowledge of syntax is used to infer word meaning.

Importantly, bootstrapping theories map sub-domains (phonology, semantics and syntax) onto other sub-domains (syntax) *within* a single mental faculty. It is reasonable to propose such theories since there is clear, *direct* interaction between the sub-domains of grammar. As reflected in Figure 1, the mental symbols of one grammatical sub-domain serve as the input or output of another. While Grammar and Number appear to have in common the property of discrete infinity, there is no known input/output relation to link these two systems formally. This is exactly what should be expected, in virtue of faculty-specificity and informational encapsulation. Rather, as in the case of other concepts, the lexicon links language to the external world of numerons, as we will discuss further below.

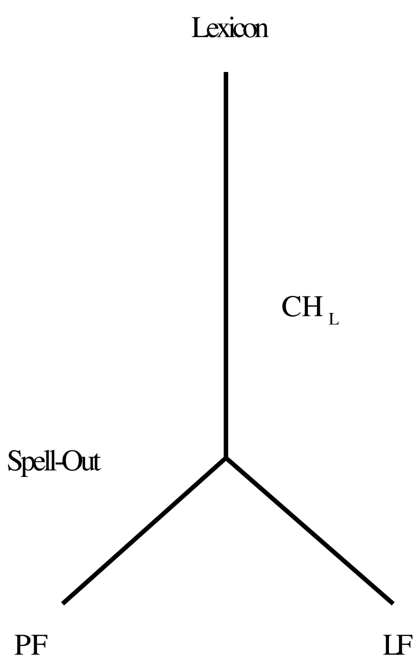

Figure 1 - Modules of Grammar in (Chomsky, 1995)

CH_L is "computational system for human language"; LF is "logical form"; PF is "phonetic form"

Notice, too, that bootstrapping theories aim to solve a problem of usable evidence. Theories of phonological bootstrapping (cf. Wanner & Gleitman (1982); Gerken, Landau, & Remez, 1990), for example, suggest that before children have a fully developed syntactic notion of word boundaries, they take advantage of existing phonetic cues in child directed speech to segment the acoustic stream into syntactic constituents. This is possible because these phonetic cues really do correspond to word boundaries. In the case of number, Bloom (1994) proposes that evidence for recursivity in the number faculty is obtained from the recursive system of grammar used when children hear others count. However, as we will now see, the linguistic count routine of English does not have a recursive definition, and consequently could not serve the purpose he has proposed.

4.3 Linguistic Counting

Even if the Number faculty were in need of evidence of recursion from grammar, even if such recursion could be bootstrapped by some as-yet-unknown operation, and even if rules and representations transferred from one faculty to another were useable, linguistic counting would provide no evidence for such recursion. For instance, in the case of American English, except where unconventional counting is used, the "... productive syntactic and morphological structures available in the counting routine ..." (Bloom, p. 186) which speakers use when counting do *not* exhibit the property of recursion. Each counting word *(one, two, three, ...)* is associated with a numeron in a one-to-one correspondence. While this set of labels is potentially infinite, it is a potentially infinite *list,* due to lexical invention, and not the result of a recursive procedure.

Bloom (1994) claims that children must go through three distinct development steps before they are able to bootstrap a generative understanding of number from the language faculty. First, in Bloom's system, children must learn that the words "one", "two" and "three" correspond to numerosities. Second, they must learn "the specifics of the linguistic counting system." Third, their non-linguistic understanding of numerosities must be mapped onto "the linguistic structure of the number system". As Bloom elaborates:

> After the mapping takes place, children can *deduce* that the number system has the property of discrete infinity by noting that there is a one-to-one correspondence between numbers and number words and coming to realize that the counting system (in a language such as English) allows for the production of an infinity of strings (one can say "a trillion", "a trillion trillion", "a trillion trillion trillion", etc.) (p. 187, emphasis in original.)

Unfortunately, Bloom does not provide a discussion of "the specifics of the linguistic count routine" or "the linguistic structure of the number system", making it difficult to evaluate the empirical content of his proposal. Upon examination of the English count routine, however, we find, in fact, that the only aspect of the system which is arguably infinite is not introduced until children have learned to count up to at least one thousand. Concretely, until the number eleven, there is not even any evidence of *regularity* in the system, much less recursion.

The English base ten counting system is idiosyncratic until it starts adding on to itself from more basic units. Generously assuming that *eleven* and *twelve* are *ten-one* and *ten-two*, respectively, at some level of linguistic abstraction, we note that the counting grammar takes another idiosyncratic turn at *thirteen*, at which point the morphology begins to be derived by adding "teen" to something like the roots of the cardinal numbers three through nine. Notice that neither of these morphological procedures can continue to apply. Hence, **tenteen* for twenty, **eleventeen* for twenty-one, **thirteenteen* for twenty-three, which would truly be recursive, are all ill-formed.[8] Twenty through ninety-nine are generally regular with *-ty* representing ten, multiplied by the number represented by the root, to which an appropriate cardinal number one through nine is added. But again, **twentyty-one* does not mean 201. So this rule is not genuinely recursive either. 100 through 999 are similarly regular, though not recursive, with the number of hundreds occurring just before the word *hundred*. Hence, "ten-hundred" can be construed to mean one thousand, but it is not part of what we call the grammar of counting in American English; similarly with "a billion billion" and "a trillion trillion."[9]

At 1000, the system finally shows much greater regularity, but it is still not genuinely recursive. The grammar of American English counting is based on a system that "tops out" at the power of 1000 (British English working somewhat differently). From the number *one* up to the number *nine hundred ninety-nine*, the word representing the cardinal value continues to increase in size, in accordance with the regular, but non-recursive, rules we have just reviewed. At *one thousand*, the grammatical representation of the cardinal value shrinks back down to two words. Then, again from the number *one thousand* up to the number *nine hundred ninety-nine thousand nine hundred ninety nine*, the term representing the cardinal value continues to increase in length until *one million*, the next power of one thousand. The English counting words continue to grow and then shrink down to two words at each successive power of 1,000, creating a sort of an "accordion effect" with respect to word size. This process can continue as long as the person counting knows, or can invent, new words for powers of 1,000.

Thus, while this system is indeed potentially infinite, the ability to count to an infinitely high number is contingent upon an ability to invent new words for powers of 1,000. This process of lexical invention, available for any new concept, is of a very different nature from recursion in grammar. The new words for powers of 1,000 cannot be inferred from previous ones; in fact, their form is essentially arbitrary, in the sense of (Saussure, 1916). This process of invention might produce an infinite list of new words, but it is not the sort of infinite list produced by syntactic or morphological recursion.

The formation of counting words in English, then, does not manifest the property of recursion, and the "productive syntactic and morphological structures available in the counting system" could not provide examples of discrete infinity which could then be "bootstrapped" or "inferred" into the number faculty.

Since there is no evidence of recursion in the linguistic count routine, one might ask if this property could be bootstrapped out of some other aspect of the grammar. There are, of course, many examples of recursion in child language, however, they occur significantly earlier than 3;6, the age at which (Wynn, 1990, 1992b) and Bloom (1994) claim children begin to count reliably.[10] For instance, in the speech of 2 Catalan-speaking children from the Serrà & Solé corpus of the CHILDES Data Base (MacWhinney & Snow, 1985; Serrà & Solé, 1986), we find examples of relativization and complementation (recursive constructions) well before the children's' 3rd birthdays, as shown in (37) and (38).

(37) *Relativization in Child Catalan* - Pep

a. Un nen dolent que hem saltat tots junts I m' ha tirat prop del peu. (Pep - 2;2.3)

a boy bad who (we) jumped together and (cl. 1st s.) has thrown close to my foot

"...a bad boy who we jumped together and he threw (null object) close to my foot."

b. Amb la Maria que s' ha caigut, que esta malaltó. (Pep - 2; 5.4)

with art. Maria who refl. cl ha fallen, who is sick.

"with Maria who fell down, who is sick."

(38) *Complementation in Child Catalan* - Gisela

a. No se, hum, no se què et donaré. (Gisela - 2;8.0)

not know (1st s.), hum, not know (1st. s) what cl. acc. (2nd s.) will give (1st s)

"I don't know, hum, I don't know what I'll give you."

b. Eh, saps que no vindrà ningú? (Gisela - 2;8.0)

eh, know (2nd s) that not will come no one?

"Hey, do you know that no one will come?"

In addition, (Gordon, Alegre, & Jackson, 1986) and (Alegre & Gordon, 1996) find evidence of recursion at the lexical level in noun compounding in children as young as 3. Given that the onset of counting does not correlate with the emergence of overt morphological or syntactic recursion in child language, it seems implausible that Bloom's bootstrapping operation could depend exclusively on the emergence of these classically recursive grammatical structures either, though Bloom himself does not propose this. Thus, accepting Bloom and Wynn's ages for the onset of reliable counting, there is no straightforward correlation between the onset of counting and the onset of grammatical processes outside of the count routine which could be characterized as discretely infinite.

Hurford (1987) provides a somewhat different formulation of the interdependence of number and language. On his view, numerical cognition is wholly derivative of the language faculty. Thus, numerical ability arises from Saussurian invention and morphosyntactic recursion. To this end, Hurford constructs a phrase structure grammar which morphologically assembles counting words, as in (39).

(39)

$$\text{NUMBER} \rightarrow \left\{\begin{array}{c}\text{DIGIT}\\ \text{PHRASE (NUMBER)}\end{array}\right\}$$

$$\text{PHRASE} \rightarrow \text{NUMBER M}$$

$$\text{M} \rightarrow \left\{\begin{array}{c}-\text{ty}\\ \text{hundred}\\ \text{thousand}\\ \text{million}\\ \text{billion}\end{array}\right\}$$

DIGIT expands to any of the words *one, two, three, ..., nine,* and context-sensitive rules convert expressions like *two-ty* to *twenty*. As Hurford notes, this procedure wildly overgenerates for any numerical expression (for instance, Hurford counts well over one million distinct structures which evaluate to the numeron 210). He thus introduces the Packing Strategy, a filter which picks out one construction from conceivably millions of candidates.

While Hurford's (1987) grammar is recursive, it does not generate labels for all the counting words, so, while its structures are in a one-to-one correspondence with the natural numbers (as are *any* two infinite sets), the semantically correct correspondences cannot hold. For instance, Hurford's grammar will not generate *trillion, vigintillion,* or *google*, for example. It will generate constructions which evaluate to over a trillion (say, "a billion billion"), but these labels do not form conventional counting words (that is, they should be ruled out by something like his Packing Strategy), contrary to Bloom's proposed examples of recursion in the counting system, cited above. Thus, Hurford's grammar generates an infinite set, but the set does not happen to be coextensive with the set of English counting words. In Hurford's system, new counting words are introduced into the phrase structure grammar in (7) by Saussurean invention -- that is, the creation of new lexical items with new semantic content.

In addition, the Packing Strategy is a psychologically implausible constraint, requiring that speakers evaluate a very large set of utterances for each numeron in order to rule out all but one (again, in the case of 210, over one million structures must be evaluated by the Packing Strategy, by Hurford's own count). Thus, Hurford's account appears implausible in light of the unrealistic computational burden it places on the mind.

To summarize, we contend that the Number Faculty is not in need of having discrete infinity transferred into it, because discrete infinity is an inherent property of the Number faculty by virtue of the recursive nature of its rules, such as the successor function. This view is supported by cases of individuals who use number, but who are unable to use grammar in a discretely infinite way. The fact that the dissociation can be found in the other direction in retarded children further bolsters the position that these are independent faculties, contra Hurford. Even if Number were not inherently recursive, discrete infinity could not be bootstrapped into it from grammar, without somehow incorporating the computations and symbols of grammar (or at least some plausible isomorph) into Number. The suggestion that the count routine could somehow serve as the grammatical source for a bootstrapping operation falls apart as soon as one carefully considers the actual grammar of the American English count routine, which lacks discrete infinity. Finally, while even the more well-known theories of bootstrapping within faculties are not uncontroversial, bootstrapping has never been proposed to take place across faculties as distinct as Number and Grammar because their points of contact are so tenuous. There is simply no analogy between what bootstrapping theories have been proposed to accomplish and what Bloom has proposed for Number and Grammar.

4.4 Inference and the Accessability of the Property of Discrete Infinity

In a revision of earlier work, Bloom (2000) argues that it is by inference and not by bootstrapping that the property of discrete infinity is transferred from the language of the count routine to the number faculty.

> Under this view, it is not that somehow children know that there is an infinity of numbers and infer that you can always produce a larger number word. Instead, they learn that one can always produce a larger number word and infer that there must therefore be an infinity of numbers. (Bloom, 2000, 238.)

As we have argued, the language of the American English count routine does not display the property of discrete infinity and thus could not serve as a source from which to transfer the property of discrete infinity, either by bootstrapping or by inference. However, even if the count routine were recursive, and even if it were possible to transfer properties without transferring the primitives and computations that produce the property, we argue that inference would still not be up to the task.

Ultimately, the property of discrete infinity in the number system is underdetermined by the evidence, similarly to the way in which discrete infinity in grammar is underdetermined by the evidence. Thus, we are dealing with a poverty of the stimulus problem analogous to the one posed by the acquisition of human language. We are able to produce and understand an infinity of utterances because the rules of grammar have an inherently discrete infinite character, <u>not</u> because we have observed an infinite number of utterances. Similarly, we are able to produce an infinite number of numerons because the successor function has an inherently discrete infinite character, and <u>not</u> because we have observed an infinite count routine. Children learning to count have no access to the successor function, but rather have access to the language-particular numerlogs of the count routine which overlay the successor function's non-verbal numeron output. Because discrete infinity is a property of the successor function and not a property of the count routine (i.e. we cannot actually observe an infinite string of count words, nor are there "productive syntactic and morphological structures available in the counting routine" which display discrete infinity), children have no explicit access to the only recursive aspect of the counting process. Consequently, the property that Bloom suggests that children should infer is completely invisible to them, making any useful inference impossible.

In summary, there is no discrete infinity in the count routine from which discrete infinity could be inferred; rather, the count routine is a linguistic convention for creating language-particular numerlogs which overlay the products of a universal rule for creating successive integers (the successor function). Furthermore, the only precise interpretation we can muster of Bloom's notion of inferring discrete infinity from the language of the count routine into the domain of numerical cognition is that computational rules and/or primitives over which these computations range must be inferred from grammar into number, which seems unlikely for the reasons given above. And finally, the idea that the property of discrete infinity could be inferred is impossible in light of the fact that the property exists as a property of the successor function, which children do not have explicit access to, as opposed to the count routine, which they do have access to.

5. Language-Number Interface Conditions

Thus far we have examined the conceptual plausibility of bootstrapping taking place across mental faculties and rejected it in principle. Furthermore, we have seen convincing empirical evidence that numerical and linguistic cognition do not depend on each other in development, but rather develop autonomously. In spite of this autonomy, however, we know that the language faculty plays a role together with number in the counting process. While we have suggested that it is logically not possible for the bootstrapping of a property to take place between Number and Grammar, we do not claim that there is no contact whatsoever between the two domains. Because the counting process recruits the resources of both domains (among others), there must exist an interface between number and grammar, as in Landau and Jackendoff's (1993) proposal regarding the interface between spatial and linguistic cognition.

In this regard, we find that the grammar of the count routine differs strikingly from the grammar of what we might call clausal syntax. The significance of this fact is that these differences likely reflect properties of the counting process. Thus, while Number and Grammar appear to be independent, they also interface and that interface is most visible in the way in which the counting process constrains the grammatical options used in the count routine.

Gelman and Gallistel (1978) and Gallistel and Gelman (1992) propose a series of principles which govern the counting process. As stated earlier, the counting process recruits resources from several faculties: number, grammar and what we will call pragmatics.[11] In what follows we will address various ways in which aspects of the number faculty itself, as one component of the counting process, influences the grammar of the count routine. We will concentrate on the Stable-Order and Cardinality Principles of Gelman and Gallistel as well as the Uniform Unit Set Condition, proposed in (Grinstead, 1996).

The Cardinality Principle

The Cardinality Principle states that the final numerical tag in the counting routine signifies the cardinality of the entire set. Notice that numerals in the count routine refer to all of the objects counted up to that point. Thus, in (40) the numeral "4" refers to all of the oranges counted up to that point. When you produce such an utterance you must have a specific set of oranges in mind. However, when the same numeral is used in a clause, as in (41), it no longer refers to a specific set of four oranges, but rather to "some set of four oranges". That is, in (41) "four oranges" simply implies the existence of four oranges, while "four oranges" in (40) implies that all of the relevant oranges under consideration have been counted.

(40) one orange..., two oranges..., three oranges..., four oranges...

(41) John ate four oranges.

Thus, (40) is referred to as Universal Quantification, while (41) is an example of Existential Quantification. Classical examples of Universal Quantification involve the use of quantifiers like 'all', 'every' and 'each', while Existential Quantifiers include words such as 'some', 'many' and the numerals. Given that each step in the counting sequence renders a cardinal value for the set calculated up to that point, it would appear that a kind of “logical” universal quantification is taking place. This is similar to what (Chomsky, 1977) argued with regard to the quantification imposed on nouns by the definite article “the.”[12] Hence, while numerals in clausal syntax are existential quantifiers, numerals in the counting routine are universal quantifiers. The Cardinality Principle demands that the final number in counting routines (and in fact every number along the way) quantify universally over the set counted up to that point. In this way, the lexical element inserted into the count routine representing the relevant numeron from the number faculty diverges from conventional clausal quantification. Thus, properties of the numbers used in the count routine are carried into the linguistic lexicon and constrain the way in which grammar may represent the count routine.

<u>The Stable Order Principle</u>

The Stable Order Principle states that the lexical tags used in the counting routine must follow a repeatable order. The result of this principle is that when counting a series of items, each one receives an ordinal position in that counting routine. The penguins in the direct object of (42) constitute an unordered set, whereas the penguins being counted in (43) each have an ordinal position in the set counted, as a result of the repeatable order imposed on the list by the Stable Order Principle.

(42) I considered the 23 penguins on the ice flow.

(43) ..., 20..., 21..., 22..., 23

Comparing the clause in (42) with the counting construction in (43) we see that the Stable Order Principle requires ordinality as well as cardinality -- a property which does not appear to exist in the rest of the grammar. Here, then, we see a property of the counting routine which simply has no analog in syntax. That is, this property is confined rather strictly to the interpretive or semantic component of the grammatical representation of the counting routine, with no syntactic reflex per se. This property is explained if we assume that a representation generated by the number module, with its faculty-specific properties, picks out semantically analogous elements in the lexicon of the grammar. However, this is wholly unexpected if we assume that the ordinal property of the counting construction is somehow generated internally by the grammar. Again, we see an instance in which a lexical item corresponding to a representation from the number faculty requires that grammar conform to its properties.

The Uniform Unit Set Condition

The Uniform Unit Set Condition of Grinstead (1996) states that the cardinality of any set can be calculated as long as the elements included in the set can be made semantically uniform. This means that when counting a homogeneous set, like pencils, the unit set is pencils, as in (44).

(44) one pencil..., two pencils..., three pencils …

When counting a heterogeneous set, however, the unit set still needs to be uniform, as when counting apples and oranges in (45).[13]

(45) one apple and orange..., two apples and oranges..., three apples and oranges...

This uniformity condition has nothing to say about the distribution of apples and oranges, but what it does mean is that the unit set for heterogeneous sets must be uniform. This continues to be true, even when the unit set is phonetically null, as in (46).

(46) one..., two..., three..., four … [counting apples and oranges]

In classifier languages, we can see this illustrated somewhat more explicitly. In Mandarin, numerical classifiers are obligatory in clausal syntax when using a numerically quantified DP. In the counting routine, however, they are optional. In the following counting routine examples there is a number, a numerical classifier and a unit set noun.

(47)	i dzi tsienbi...,	iang dzi tsienbi...,	san dzi tsienbi...,	si dzi tsienbi
	one CL. pencil...	two CL. pencil...	three CL. pencil...	four CL. pencil
	one pencil...	two pencils...	three pencils...	four pencils'

If, however, one attempts to count both pencils and books, the latter of which takes a different numeral classifier for "volumes," counting cannot proceed with a number, a mixed unit set like "pencils and books" and some numerical classifier. Numerical classifiers cannot modify semantically mixed unit sets.

(48) * i dzi/ben/ge tsienbi han shu...	*liang dzi/ben/ge tsienbi han shu...
one (pnc. CL.)/(vol. CL.)/(gen. CL.) pencil and book...	two (pnc. CL.)/(vol. CL.)/(gen. CL.) pencil and book...
'one book and pencil...	two books and pencils'

However, if someone is instructed to count "the objects" on the top of a desk and the objects are pencils and books, counting will either proceed with numerals only or with numerals and a general classifier for objects.

(49)	i (ge)...	liang (ge)...	san (ge)...
	one (gen. CL.)	two (gen. CL.)	three (gen. CL.)

We suggest that this default classifier is evidence that the uniform unit set of the number faculty must be represented in grammar in a similarly uniform way, whether this representation is grammatically reflected by the unit set noun only, as in English, or by either a numeral classifier or unit set noun, as in Mandarin.

The Uniform Unit Set Condition, then, is a property of the counting process in that only semantically like objects can be included in a set when calculating its cardinality. When counting objects which are not alike in semantically significant respects, such as those instantiated in the numeral classifier types, the grammar recruits semantically more general expressions to count the non-uniform objects as "things" and uses a semantically general or default classifier.[14] Under our assumptions, this takes place when a lexical item within the language faculty is linked with a numerical representation in the conceptual domain, generated by the number faculty. Properties of the number faculty, such as those discussed in this section, are reflected in the grammatical options chosen for the representation of counting.

Summarizing, then, we have seen that, in the count routine, universal quantification, set ordinality and uniformity of the unit set are numerical properties reflected in grammar and correspond to the Cardinality and Stable-Order Principles and the Uniform Unit Set Condition, and result from interface among Number, Grammar and Pragmatics in the counting process. There appears to be no uniquely linguistic reason that these properties should arise in the grammar of the count routine. They follow logically, however, within our framework of a lexically mediated interface between number and grammar.[15]

6. The Lexical Interface Between Number and Grammar

It appears correct that the computations of numerical and grammatical cognition are not dependent on each other for either in their existence in the adult state or for their development, and yet we have seen that the nature of the lexical elements of the Number faculty and the way they are deployed in the count routine effects the grammatical representation of numbers in the count routine.

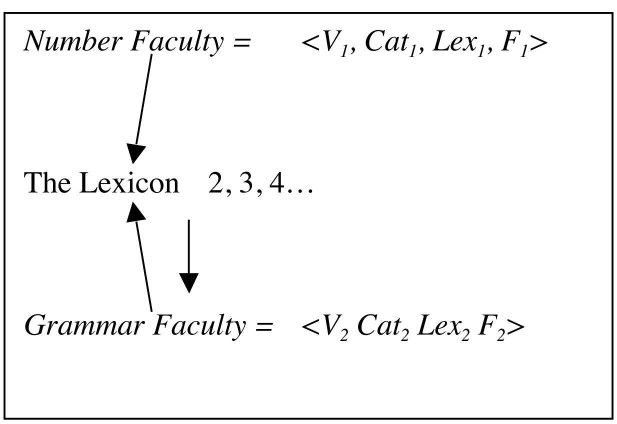

Figure 2 - The Lexical Interface Between the Number and Grammar Faculties

Our proposal, then, is that the lexicon is the interface between Number and Grammar, as represented in Figure 2. In this view, the Number Faculty provides a conceptual representation which can be lexicalized as a quantifier, among other grammatical categories, in the Lexicon. The Lexicon then makes this grammaticalized representation of a number available to the Grammar Faculty for use in its derivations. In the same way that locatives such as *between*, *through* or *near* encode certain geometrical, spatial relations, determined in the domain of the mind dedicated to spatial cognition, numbers in the linguistic count routine encode properties which are determined in the Number faculty. The semantic properties which originate in the Number faculty, discussed in section 5 (Universal Quantification, Set Ordinality & Semantic Uniformity of counted objects), presumably obtain in all languages, while the particular grammatical structures of the count routine may vary by language. Thus, the English counting system is a base-10 system, while others are not (French, for example).

In this view, the lexicon of each particular language will limit the grammatical representation of counting in such a way that these presumably universal semantic counting properties are represented. However, the language-particular, idiosyncratic means of constructing a linguistic count routine, outside of these universal properties, will have some latitude for variation. Thus, English marks plurality on nouns, so the lexicon must assure that count expressions respect this fact by including the plural morpheme when counting above one and by leaving it off when counting using the number one. Mandarin, on the other hand, does not mark plurality on nouns and consequently its lexicon will not be concerned with plurality, but rather with classifiers, as discussed in section 5.[16]

7. Conclusion

We have argued that the number faculty is neither derivative of the system of grammar, as Hurford maintains, nor does its development depend upon any bootstrapping or inferential relation with the language faculty, as Bloom claims. Moreover, the notion of cross-faculty bootstrapping is difficult to interpret, given the inexplicit terms in which it has been discussed. While it is implausible that properties of rule systems transfer from one domain to another divorced from the rules and representations that they characterize, it seems even less plausible to suggest that rules and representations could be transferred directly from one domain to another in the light of domain-specific conceptions of mental architecture, proposed in much modern theorizing, supported by cases presented in this article. Moreover, contrary to Bloom's claims, the grammar of the English count routine does not offer an example of recursion from which discrete infinity could be bootstrapped or inferred. Finally, if Number depended on Grammar in either of the ways suggested by Bloom or Hurford, the developmental autonomy of the two domains found in the cases presented should not exist.

While we conclude that rules and representations of the two domains are not directly commensurable, it appears that language and number indirectly interface at the level of conceptual structure, where lexical items are paired with meanings. On this view, cognitive architecture allows insertion of a lexical items into the computational system of grammar which, albeit indirectly, can carry <u>semantic</u> properties particular to the numerical domain with it. Thus, the grammatical lexicon allows quantifiers and nouns to serve as proxies for representations generated in the numerical domain. While this "lexical commensurability" can plausibly hold between lexical items in the two domains, nothing in this relationship suggests discrete infinity and consequently does not lend itself to the putative bootstrapping or inferential relation proposed by Bloom. The plausibly universal semantic properties of the counting process (Universal Quantification, Ordinality and Set Uniformity) constrain the lexicons of particular languages, which represent these properties by means of their language-particular grammar.

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Endnotes

[1]There was always a marked scatter in her scores, given her selective verbal ability. At age 11;9,for example, her WISC-R verbal IQ was 58; her Performance IQ 0! (Yamada, 1990).

[2]Chelsea has learned a rule for marking plural on nouns and does so, although not consistently. However, she has no definite determiners, thus her plurals are not a reflex of grammatical agreement. In addition, subject-verb number agreement marking is neither comprehended nor produced, nor is number marking on pronouns.

[3] Bloom (2000) states "Note that this theory has no problem with people with severe language impairments who might have a rich generative understanding of number. The claim here is not that numerical understanding emerges from language in general but that it emerges from learning the system of number words." (p. 236). As we show in section 4.3, the system of number words or count routine of American English is not a generative system and thus could not be the source of a "rich generative understanding of number".

[4]Notice that this particular function within the number faculty only applies to certain of the faculty's lexical items, namely, the natural numbers. It cannot, for example, compute the successor of a fraction, given the infinite number of numbers between any given pair of numbers. See (Hartnett, 1991) for further discussion.

[5]While a possible linguistic grammar could compute a successor for natural numbers (such as a concatenation grammar), such grammars cannot represent natural languages and are therefore not relevant to our claims.

[6] The system of inputs and outputs represented in Figure 1 are not intended to imply real-time processing, as in performance models (compare Levelt, 1989), but rather represent derivational steps in the theory of linguistic competence.

[8] The fact that children produce just such utterances in an attempt to *regularize* an irregular system is evidence of the lack of regularity and discrete infinity in the grammar of the count routine.

[9] In this section we will address the American English count routine, which does not include numbers formed by iterative modification, of the kind which Bloom incorrectly asserts can form part of the count routine, in his comment which states that "…the counting system (in a language such as English) allows for the production of an infinity of strings (one can say "a trillion", "a trillion trillion", "a trillion trillion trillion, etc." (p. 187). Such strings are as ill-formed in the American English count routine as are child productions of the "a hundred hundred" or "twenty-ten" variety, though they may sound more plausible because, outside of astronomy, most people do not deal with numbers this large. The same, of course, can be said of children who have not yet had to deal with a thousand ("a hundred hundred").

[10]See (Gelman & Meck, 1992) and (Gelman, 1993) for evidence that children in fact reliably count well before 3;6.

[11](Gelman & Greeno, 1989) refer to this as "interpretive competence".

[12]Assuming a theory of Determiner Phrases for counting expressions, Grinstead (1996) contends that the unit set nouns in the counting routine occur in the D_0 position, as do Proper names and other logically universal quantifiers, according to (Longobardi, 1994a), and 1st and 2nd person pronouns, according to (Abney, 1987). The idea, then, is

that the unit set noun has the semantic property of being universally quantified and grammar expresses this property by building the unit set noun into the phrase structure position that allows universal quantification to be expressed.

[13] Notice that "*one apples and oranges" is ill-formed, suggesting that there is a grammatical agreement relation here, as in clausal syntax. Notice also that at the first step in this counting routine we must be pointing at either an apple or an orange only (by definition) and yet the unit set must be "apples and oranges," otherwise we are not counting a uniform unit set. This again implies the necessity of a semantically homogeneous unit set.

[14] We are grateful to Chai-shune Hsu, Benjamin Wang, Tetsuya Sano and Motoko Ueyama for their help with the classifier facts.

[15] We observe that Gelman and Gallistel's (1978) One-to-One Principle has the grammatical consequence that all the nouns in the count routine must be definite, as opposed to non-specific or specific indefinite, as allowed in clausal syntax. This is a reflex of deixis, which is part of the performance system of counting, which we wish to distinguish from the Cardinality and Stable Order Principles of Gelman and Gallistel, which characterize properties particular to the number faculty. For more on the grammatical reflexes of the One-to-One Principle, see Grinstead (1996).

[16] Interestingly, (Castillo-Berastegui, 2001) suggests that classifiers and plural marking are the same grammatical element.